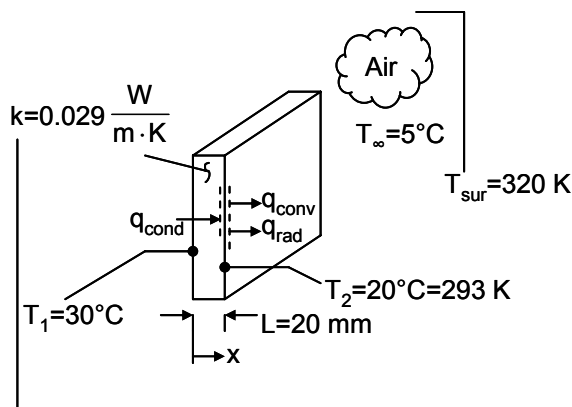


PROBLEM 1.75

KNOWN: Thermal conductivity, thickness and temperature difference across a sheet of rigid extruded insulation. Cold wall temperature, surroundings temperature, ambient temperature and emissivity.

FIND: (a) The value of the convection heat transfer coefficient on the cold wall side in units of $\text{W}/\text{m}^2\cdot^\circ\text{C}$ or $\text{W}/\text{m}^2\cdot\text{K}$, and, (b) The cold wall surface temperature for emissivities over the range $0.05 \leq \varepsilon \leq 0.95$ for a hot wall temperature of $T_1 = 30^\circ\text{C}$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (c) Constant properties, (4) Large surroundings.

ANALYSIS:

(a) An energy balance on the control surface shown in the schematic yields

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}} \quad \text{or} \quad q_{\text{cond}} = q_{\text{conv}} + q_{\text{rad}}$$

Substituting from Fourier's law, Newton's law of cooling, and Eq. 1.7 yields

$$k \frac{T_1 - T_2}{L} = h(T_2 - T_\infty) + \varepsilon\sigma(T_2^4 - T_{\text{sur}}^4) \quad (1)$$

or
$$h = \frac{1}{(T_2 - T_\infty)} \left[k \frac{T_1 - T_2}{L} - \varepsilon\sigma(T_2^4 - T_{\text{sur}}^4) \right]$$

Substituting values,

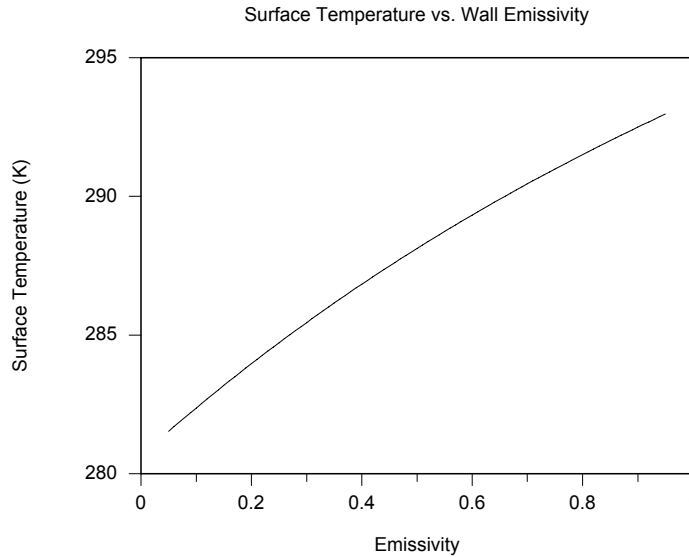
$$h = \frac{1}{(20 - 5)^\circ\text{C}} \left[0.029 \frac{\text{W}}{\text{m} \cdot \text{K}} \times \frac{(30 - 20)^\circ\text{C}}{0.02 \text{ m}} - 0.95 \times 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} (293^4 - 320^4) \text{ K}^4 \right]$$

$$h = 12.2 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \quad <$$

Continued...

PROBLEM 1.75 (Cont.)

(b) Equation (1) may be solved iteratively to find T_2 for any emissivity ϵ . *IHT* was used for this purpose, yielding the following.



COMMENTS: (1) Note that as the wall emissivity increases, the surface temperature increases since the surroundings temperature is relatively hot. (2) The *IHT* code used in part (b) is shown below. (3) It is a good habit to work in temperature units of kelvins when radiation heat transfer is included in the solution of the problem.

//Problem 1.75

$h = 12.2$ //W/m²·K (convection coefficient)
 $L = 0.02$ //m (sheet thickness)
 $k = 0.029$ //W/m·K (thermal conductivity)
 $T_1 = 30 + 273$ //K (hot wall temperature)
 $T_{sur} = 320$ //K (surroundings temperature)
 $\sigma = 5.67 \times 10^{-8}$ //W/m²·K⁴ (Stefan -Boltzmann constant)
 $T_{inf} = 5 + 273$ //K (ambient temperature)
 $e = 0.95$ //emissivity

//Equation (1) is

$$k(T_1 - T_2)/L = h(T_2 - T_{inf}) + e\sigma(T_2^4 - T_{sur}^4)$$